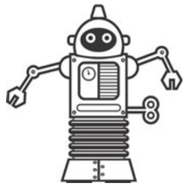


Robot Art

TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
What set of instructions should you give to a robot so that it can draw a certain type of picture?

LEARNING GOALS
Students will:

- Use numeric, verbal, and visual data to instruct a robot to draw a given picture
- Consider how to use and communicate data
- Make decisions about whether or not a solution meets the needs of a client
- Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards. Please see pages 4-7 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

- Overhead transparencies and transparency markers/pens, whiteboards and markers, posterboards, or other presentation tools such as a document camera.
- Calculators
- Rulers, scissors, tape
- Markers, colored pencils, pencils
- Construction paper, graph paper, lined paper
- Paper towels or tissues (for cleaning transparencies)
- Manila folders or paper clips for collecting the students' work
- Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students' solutions to the task reveal explicitly how they are thinking about the given situation.

THE ROBOT ART MEA CONSISTS OF FIVE COMPONENTS:
1) **Additional information sheet:** This sheet explains in additional detail the specifics of this MEA, including the description and instructions for an optional pre-MEA activity, specific supplies needed for this MEA, and an optional post-MEA activity. This handout is on page 8.
2) **Newspaper article:** Students individually read the newspaper article to become familiar with the context of the problem. This handout is on page 9.
3) **Readiness questions:** Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 10
4) **Problem statement:** In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students' first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 11-12. Each team needs the handouts on pages 11-12.

5) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, MEA study takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 15.

RECOMMENDED PROGRESSION OF THE ROBOT ART MEA

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills' discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

Social Skills (3 -5 minutes)

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision- making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA.
(http://www.youtube.com/user/flowmathematics)

Newspaper Article and Readiness Questions:
The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

Problem Statement:

You may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group's solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

Working on the Problem Statement (35-50 minutes)**:** Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

Teachers' role: As they work, your role should be one of a facilitator and observer.

Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (15-30 minutes)**:** The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams' presentations but also to a) ***try to understand the other teams' solutions*** and b) ***consider how well these other solutions meet the needs of the client.*** You may want to offer points to students that ask 'good' questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 16 in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams' memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS' WORK

You can decide if you wish to evaluate the students' work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client's needs?

Requires redirection*:* The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements*:* The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions*:* The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions*:* No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable: The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME

You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won't be answering questions such as "Is this the right way to do it?" or "Are we done yet?" You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE ROBOT ART MEA

You may find the Observation Form (page 14 useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out "real-time" as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 15. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Robot Art MEA after hearing of the other teams' solutions.

STUDENT REFLECTION FORM

You may find the Student Reflection Form (page 16) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED

NCTM MATHEMATICS STANDARDS

Numbers and Operations:

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand and use ratios and proportions to represent quantitative relationships
- Judge the reasonableness of numerical computations and their results
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods
- Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Algebra

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Relate and compare different forms of representation for a relationship
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations

- Use symbolic algebra to represent and explain mathematical relationships
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
- Draw reasonable conclusions about a situation being modeled

Geometry

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
- Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
- Use coordinate geometry to represent and examine the properties of geometric shapes
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Measurement

- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume
- Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision
- Solve problems involving scale factors, using ratio and proportion

Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

- Develop and evaluate mathematical arguments and proofs

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Use representations to model and interpret physical, social, and mathematical phenomena

NRC Science Standards

Inquiry

- Use appropriate tools and techniques to gather, analyze and interpret data
- Develop descriptions, explanations, predictions, and models using evidence
- Think critically and logically to make the relationships between evidence and explanations
- Recognize and analyze alternative explanations and predictions
- Communicate scientific procedures and explanations
- Use mathematics in all aspects of scientific inquiry

Common Core State Standards

- K.G: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
  1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

2. Correctly name shapes regardless of their orientations or overall size.

3. Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and

vertices/“corners”) and other attributes (e.g., having sides of equal length).

5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

6. Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”

- 1.G Reason with shapes and their attributes.
  1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

  2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.4

  3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

- 2.MD Measure and estimate lengths in standard units.
  1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

  2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

  3. Estimate lengths using units of inches, feet, centimeters, and meters.

  4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

- 2.G Reason with shapes and their attributes.
  1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

- 3.G Reason with shapes and their attributes.
  2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

- 4.G: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
  1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

  2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

- 5G: Classify two-dimensional figures into categories based on their properties
  3. Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

4. Classify two-dimensional figures in a hierarchy based on properties.

Standards for Mathematical Practices integration with MEAs

Mathematical Practice	How it occurs in MEAs
1. Make sense of problems and persevere in solving them.	As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.
2. Reason abstractly and quantitatively	MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.
3. Construct viable arguments and critique the reasoning of others.	Throughout MEAs while groups are working and presenting their models.
4. Model with mathematics.	This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.
5. Use appropriate tools strategically.	Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.
6. Attend to precision.	Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.
7. Look for and make use of structure.	Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.
8. Look for and express regularity in repeated reasoning.	As participants develop their models the patterns they notice can assist in their model development.

Robot Art MEA

Optional Pre-Activity – Robot / Computer Directions (Peanut Butter Sandwich Activity)

This is a simple, commonly used activity to help students understand that computers only do exactly what they are told and that computer programs have to make every single step explicit. Students are provided with supplies to make a peanut butter and jelly sandwich (loaf of bread, knife, jar of peanut butter, jar of jelly – be sure to check for allergies before the activity). One student is the sandwich maker and another is the director. The sandwich maker is asked to do exactly what the director tells them and nothing else. The director is told to tell the sandwich maker how to make a peanut butter and jelly sandwich. The need for explicit directions and absurdity of what happens without them becomes quickly evident. You can pair students for the activity or have volunteers show the entire class. Variations also include having the director blind folded. You can access directions for the activity as well as help with supporting class discussion at numerous web sites including http://downloads.cas.psu.edu/4H/ActivitySheet42.

Pre-Reading – Robot Art / “Robot Art” Questions

The article and questions for “Robot Art” help to generate student interest in and orientate students to the concept of robots that make art.

Model-Eliciting Activity – Telling a Robot Artist How to Draw

This is the main model eliciting activity. Students should work in small groups (3 or 4 students). Groups should be given a time limit for making their instructions (recommended at least one hour). All groups should be prepared to and asked to share their solution with the entire class at the end of the activity. Allow students to ask questions and comment on other groups’ ideas, but be sure the discussion remains respectful.

Supplies for Model-Eliciting Activity – sample pictures from packet in envelopes; graph paper with small (less than 1cm) squares, gridded paper with large (greater than 2cm) squares, plain paper, rulers, compasses

Robot Art

from New Scientist (December 23, 2006 – 02624079)



Max Chandler’s art studio is more chaotic than most. It's not that he is disorganized: the mess is down to the team of brush-wielding robots that help him paint his pictures. Built from Lego bricks and aluminum, the bots look like miniature trucks and cranes. Some propel themselves on wheels, while others shuffle about on legs. All are capable of frustrating behaviors that sometimes turn peaceful painting sessions into circus-like confusion.

If the robots are not tracking paint onto the neighbor's porch, they're falling clean off the canvas, ruining the flow of the picture. Worse, they always need paint refills in the middle of the picture, forcing Chandler to scramble to insert a new paint-loaded brush. "Sometimes I think this is the worst way in the world to create a picture," he chuckles.

So why go to all this trouble? Although the robots make life more difficult, Chandler says working with them is the only way he can create the pictures he wants. Chandler's intention has always been to infuse his art with mathematics. An avid painter since childhood, he has loved math for almost as long: at nine, he was already writing computer programs. He went on to study math at MIT and has paid the bills working as a programmer. "I have always had this dual nature of being tied up in science and in art," he says. Simply rendering mathematical patterns on a computer and printing them out created boring and repetitive pictures. To get the shapes he wanted, he could program small robots to drag a paint brush or pen as they moved.

You might wonder if Chandler feels the end result is worth all the effort. His answer is a resounding yes. "It wouldn't be the same if it was just the robot or just me," he says. "Even if a viewer doesn't know why it is, the picture will look different." Whether the robots create something better than a human could is subjective, of course, but the pictures are unique. People say, “Wow, those lines are special -- how did you do that?"

"Robot Art" Questions

1) How is Max Chandler's art different from a lot of other art?

2) Why does he use robots instead of a computer and printer?

3) What did Max Chandler study at college at MIT? How does he use this knowledge as an artist?

Telling a Robot Artist How to Draw

For the National Robotics Challenge, there is a special competition to design and build an Art Robot. A team from the local high school has built a robot that can draw dots, lines, and common shapes like circles, arches, squares, triangles, ovals, trapezoids, and rectangles. The robot works great, but the team is having problems getting the robot to make "art" rather than just draw dots, lines, and shapes.

They think it would be cool to have the robot recreate a picture like those below by following spoken directions from someone. They plan to ask one of the judges to draw a picture and then one of the high school team members will tell the robot how to draw its own version of that picture.

The high school team is working hard on reprogramming the robot to accept spoken directions, but they need your help to know what directions they should say to the robot to make pictures like these. They have asked your team for help finding out how to tell a robot to draw its own version of a picture like the one below.

Your team has agreed to help the high school robotics team. Your team will make a set of instructions explaining how to tell a robot to draw a picture. The high school robotics team has given your team the following information to help you understand how a robot "thinks" and what they need from you:

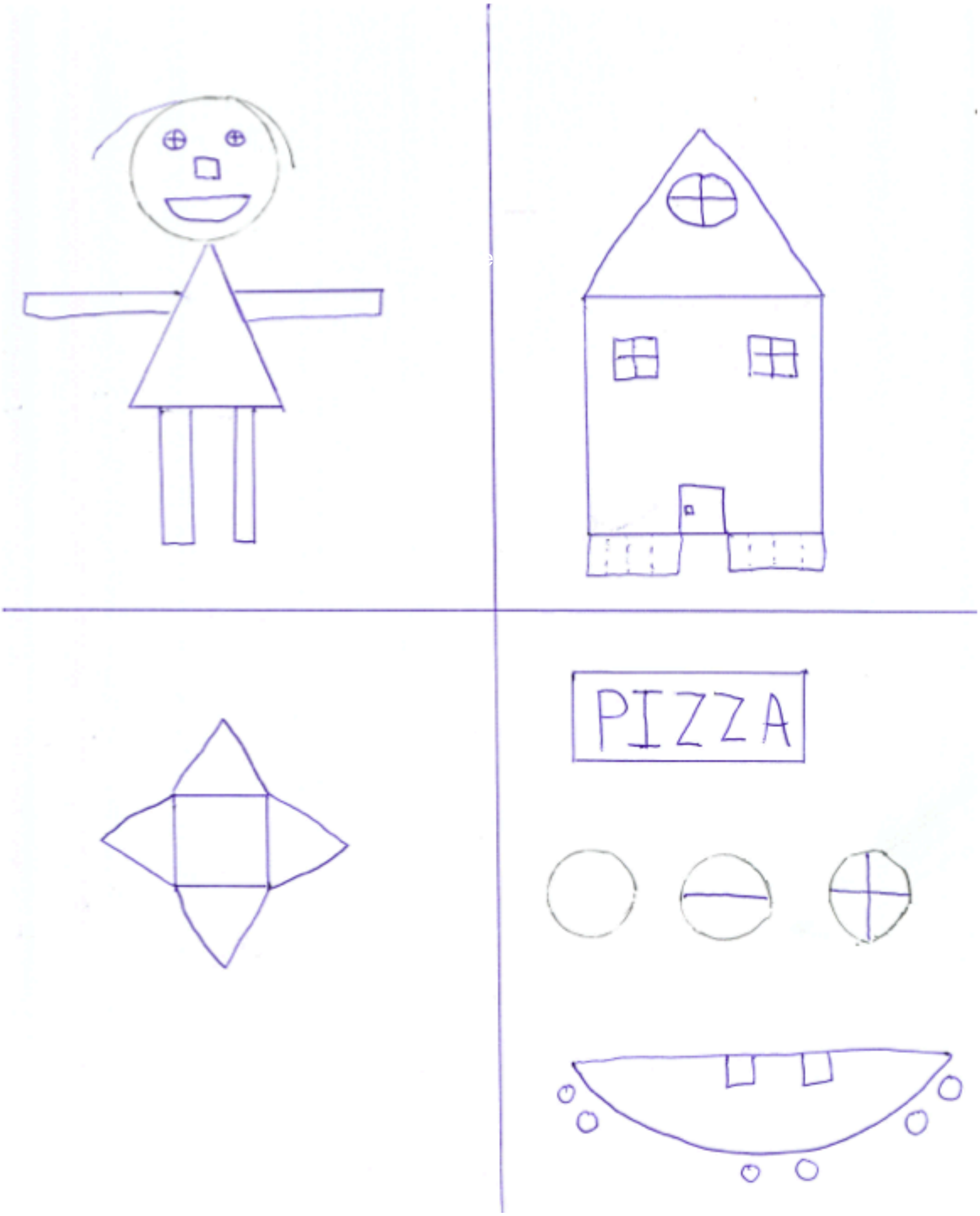
1) One person on your team should be able to look at a picture and tell another person on your team how to draw a picture even though the person drawing has never seen the picture. There are sample pictures in the envelope provided so you can try this out in your group.

2) The person drawing the picture is pretending to be the robot; so, they have to follow directions exactly as they are given. If just asked to draw a circle, a robot would not know where to draw the circle or how big to make it. All instructions need to be very specific.

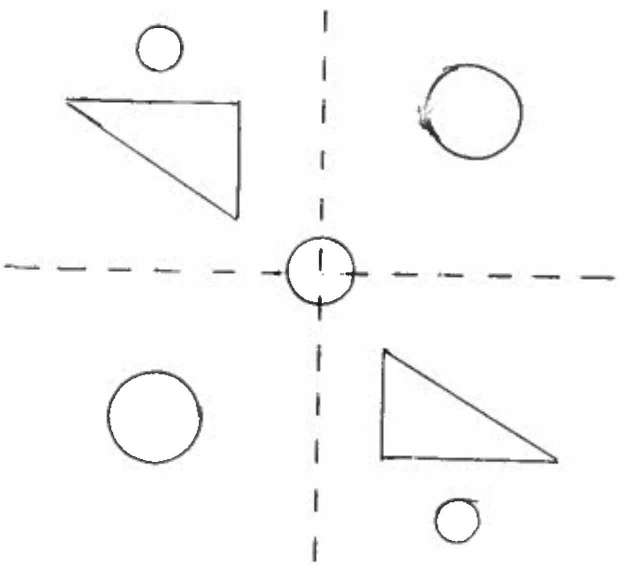
3) We want the robot to truly be an "artist" able to draw anything (like a picture made by a judge); so, we need instructions for how to tell the person drawing or robot about any picture not just one particular picture.

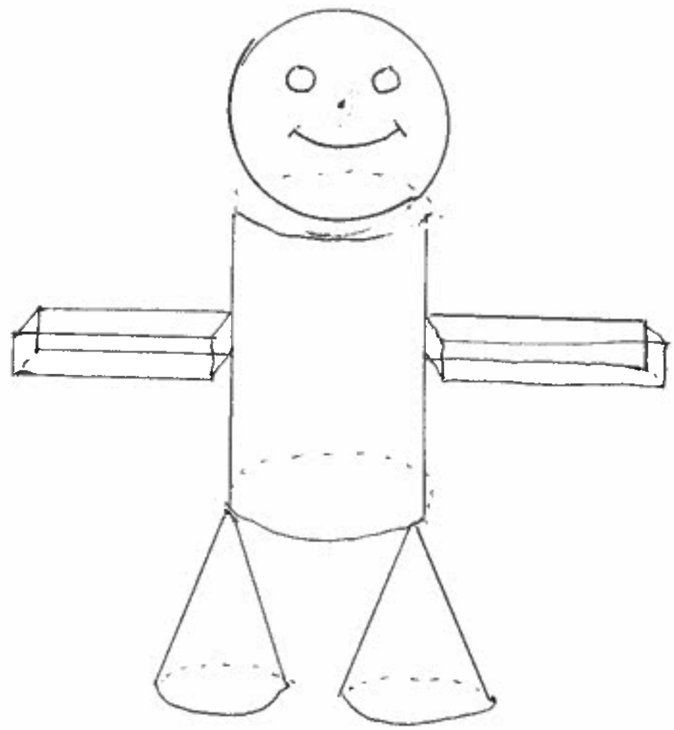
4) The new pictures need to look like the original picture but do not have to be exactly the same. Our robot is an artist with its own style.

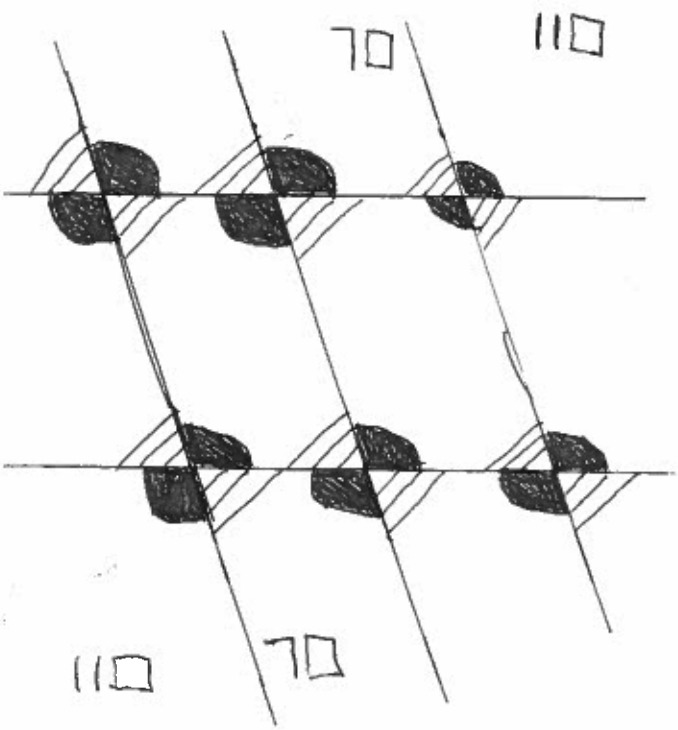
Your team needs to make instructions describing what a robot should be told so that it draws a picture. Your team will present these instructions to the high school robotics team to help them understand what to do at their competition.

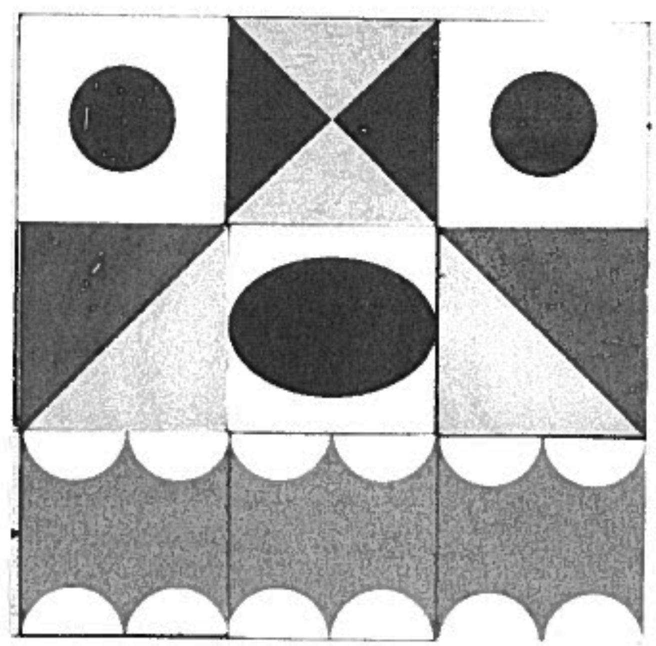

PIZZA

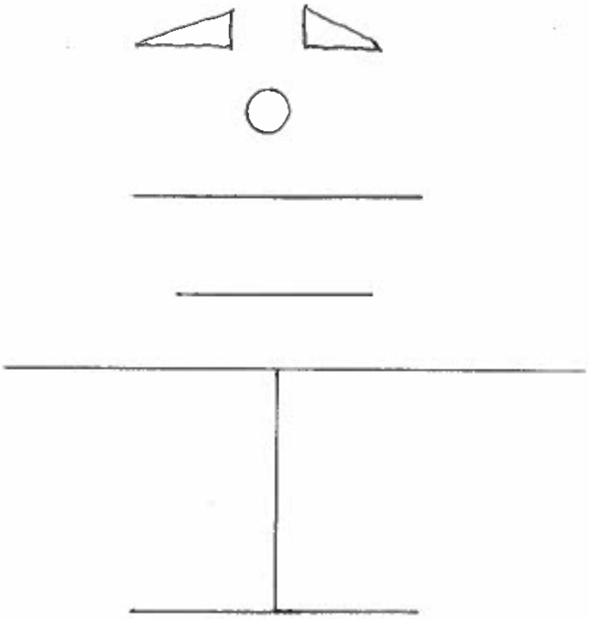
Pictures for Middle School Students

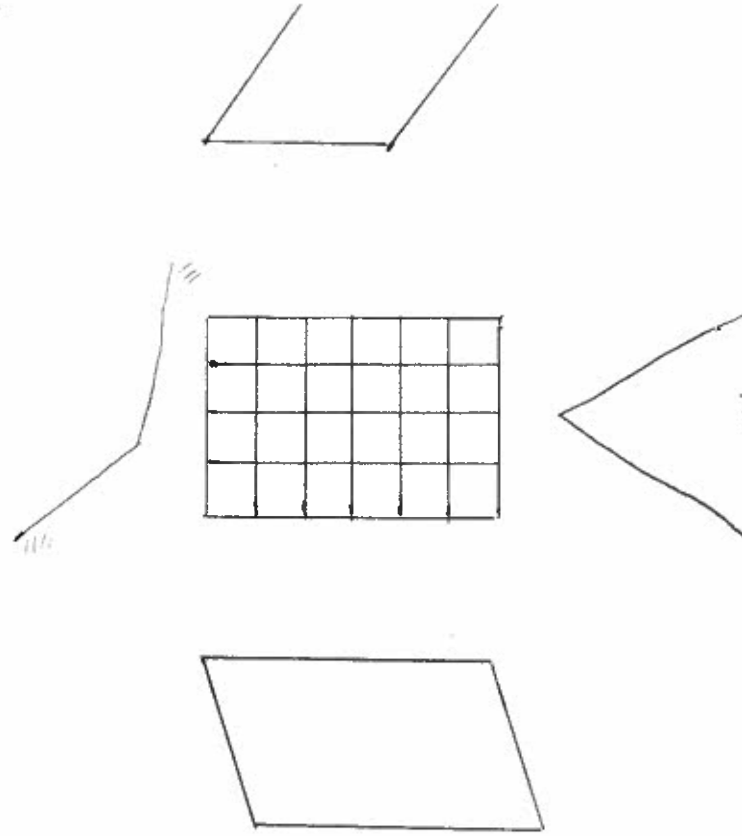


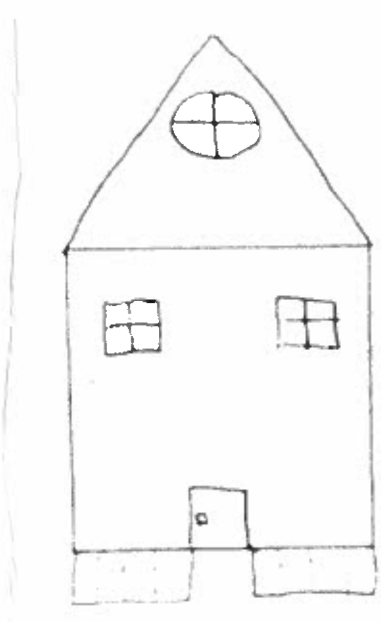


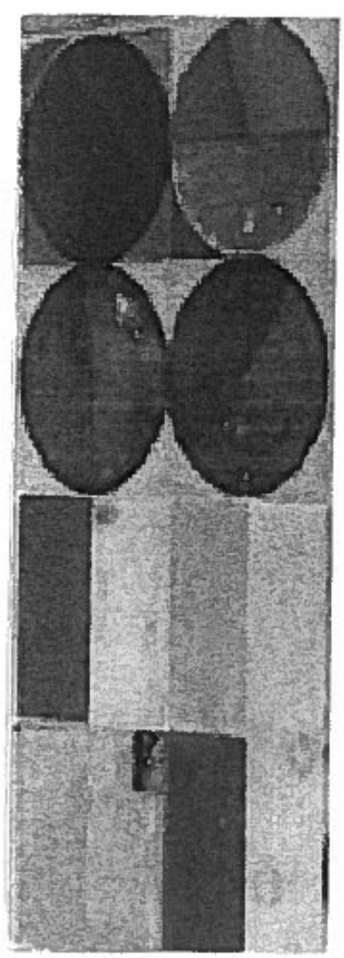












OBSERVATION FORM FOR THE TEACHER- Robot Art MEA

Team: ___________________________________

Math Concepts Used:
What mathematical concepts and skills did the students use to solve the problem?

Team Interactions:
How did the students interact within their team or share insights with each other?

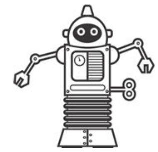
Data Organization & Problem Perspective:
How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:
What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:
How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?

PRESENTATION FORM – Robot Art MEA



Name____

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, "They should have organized their presentation better." Evaluate their solution only.

Team ____________________________

What I liked about their solution:

What I didn't like about their solution:

Team ____________________________

What I liked about their solution:

What I didn't like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.

STUDENT REFLECTION FORM – Robot Art MEA

Name ______________________________ Date______________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?

Not at all A little bit Some Most of it All of it

Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?