Trash Trouble

TOPIC
Mathematical Connections and Problem Solving

KEY QUESTION
How do you create a procedure or formula for predicting the amount of trash the U.S. will produce in the year 2020?

LEARNING GOALS
Students will:

- Use numeric data to create an accurate predication.
- Understand what causes the amount of trash to be increased and how recycling can help.
- Make decisions about whether or not a solution meets the needs of a client
- Communicate the solution clearly to the client

GUIDING DOCUMENTS
This activity has the potential to address many mathematics and science standards, as well as address engineering principles. Please see pages 4-6 for a complete list of mathematics and science standards.

RECOMMENDED SUPPLIES FOR ALL MODEL-ELICITING ACTIVITIES
It is recommended to have all of these supplies in a central location in the room. It is recommended to let the students know that they are available, but not to encourage them to use anything in particular.

- Overhead transparencies and transparency markers/pens or whiteboards and markers, posterboards, or other presentation tools such as a document camera.
- Calculators
- Rulers, scissors, tape
- Markers, colored pencils, pencils
- Construction paper, graph paper, lined paper
- Paper towels or tissues (for cleaning transparencies)
- Manila folders or paper clips for collecting the students' work
- Optional: Computers with programs such as Microsoft Word and Excel

WHAT ARE MODEL-ELICITING ACTIVITIES (MEAs)?
Model-Eliciting Activities are problem activities explicitly designed to help students develop conceptual foundations for deeper and higher order ideas in mathematics, science, engineering, and other disciplines. Each task asks students to mathematically interpret a complex real-world situation and requires the formation of a mathematical description, procedure, or method for the purpose of making a decision for a realistic client. Because teams of students are producing a description, procedure, or method (instead of a one-word or one-number answer), students' solutions to the task reveal explicitly how they are thinking about the given situation.

THE Trash Trouble MEA CONSISTS OF FOUR COMPONENTS:
1) **Newspaper article:** Students individually read the newspaper article to become familiar with the context of the problem. This handout is on pages 7-8.
2) **Readiness questions:** Students individually answer these reading comprehension questions about the newspaper article to become even more familiar with the context and beginning thinking about the problem. This handout is on page 9.
3) **Problem statement:** In teams of three or four, students work on the problem statement for 45 – 90 minutes. This time range depends on the amount of self-reflection and revision you want the students to do. It can be shorter if you are looking for students' first thoughts, and can be longer if you expect a polished solution and well-written letter. The handouts are on pages 10-15. You decide if you want to also provide them with the graph on page 12.

4) **Process of sharing solutions:** Each team writes their solution in a letter or memo to the client. Then, each team presents their solution to the class. Whole class discussion is intermingled with these presentations to discuss the different solutions, the mathematics involved, and the effectiveness of the different solutions in meeting the needs of the client.

In totality, each MEA takes approximately 2-3 class periods to implement, but can be shortened by having students do the individual work during out-of-class time. The Presentation Form can be useful and is explained on page 4 and found on page 17.

RECOMMENDED PROGRESSION OF THE Trash Trouble MEA

While other implementation options are possible for MEAs, it is recommended that the MEA be implemented in a cooperative learning format. Numerous research studies have proven cooperative learning to be effective at improving student achievement, understanding, and problem solving skills. In this method students will complete work individually (Newspaper article and readiness questions; as well as initial thoughts on the problem statement) and then work together as a group. This is important because brainstorming works best when students have individual time to think before working as a group. Students can be graded on both their individual and group contributions. Social skills' discussion at the beginning of the MEA and reflection questions at the end of the MEA are also essential aspects of cooperative learning.

Social Skills (3 -5 minutes)

Students must be taught how to communicate and work well in groups. Several social skills that are essential to group work are decision- making, asking questions, and communicating and listening. The teacher can show part of a YouTube video and discuss aspects of these skills before beginning the MEA. (http://www.youtube.com/user/flowmathematics)

Newspaper Article and Readiness Questions:

The purpose of the newspaper article and the readiness questions is to introduce the students to the context of the problem.

(10 minutes): Give the article and the questions to the students the day before for homework. Then, in the next class, discuss as a class the answers to the readiness questions before beginning to discuss the problem statement.

Problem Statement:

You may want to read the problem statement to the students and then identify as a class: a) **the client that the students are working for** and b) **the product that the students are being asked to produce**. Once you have addressed the points above, allow the students to work on the problem statement. Let the students know that they will be sharing their solution to the rest of the class. Tell students you that you will randomly pick a group member to present for each group. Tell the students that they need to make sure that everyone understands their group's solution so they need to be sure to work together well. The group member who will present can be picked by assigning each group member a number.

Working on the Problem Statement (35-50 minutes)**:** Place the students in teams of three or four. Students should begin to work by sharing their initial ideas for solving the problem. If you already use teams in your classroom, it is best if you continue with these same teams since results for MEAs are better when the students have already developed a working relationship with their team members. If you do not use teams in your classroom and classroom management is an issue, the teacher may form the teams. If classroom management is not an issue, the students may form their own teams. You may want to have the students choose a name for their team to promote unity.

Teachers' role: As they work, your role should be one of a facilitator and observer. Avoid questions or comments that steer the students toward a particular solution. Try to answer their questions with questions so that the student teams figure out their own issues. Also during this time, try to get a sense of how the students are solving the problem so that you can ask them questions about their solutions during their presentations.

Presentations of Solutions (15-30 minutes)**:** The teams present their solutions to the class. There are several options of how you do this. Doing this electronically or assigning students to give feedback as out-of-class work can lessen the time spent on presentations. If you choose to do this in class, which offers the chance for the richest discussions, the following are recommendations for implementation. Each presentation typically takes 3 – 5 minutes. You may want to limit the number of presentations to five or six or limit the number of presentations to the number of original (or significantly different) solutions to the MEA.

Before beginning the presentations, encourage the other students to not only listen to the other teams' presentations but also to a) ***try to understand the other teams' solutions*** and b) ***consider how well these other solutions meet the needs of the client.*** You may want to offer points to students that ask 'good' questions of the other teams, or you may want students to complete a reflection page (explanation – page 4, form – page 17) in which they explain how they would revise their solution after hearing about the other solutions. As students offer their presentations and ask questions, whole class discussions should be intermixed with the presentations in order to address conflicts or differences in solutions. When the presentations are over, collect the student teams' memos/letters, presentation overheads, and any other work you would like to look over or assess.

ASSESSMENT OF STUDENTS' WORK

You can decide if you wish to evaluate the students' work. If you decide to do so, you may find the following Assessment Guide Rubric helpful:

Performance Level Effectiveness: Does the solution meet the client's needs?

Requires redirection*:* The product is on the wrong track. Working longer or harder with this approach will not work. The students may need additional feedback from the teacher.

Requires major extensions or refinements*:* The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.

Requires editing and revisions*:* The product is on a good track to be used. It still needs modifications, additions or refinements.

Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions*:* No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR Small changes needed to meet the generalized needs of the client.

Share-able or re-usable**:** The tool not only works for the immediate solution, but it would be easy for others to modify and use in similar situations. OR The solution goes above and beyond meeting the immediate needs of the client.

IMPLEMENTING AN MEA WITH STUDENTS FOR THE FIRST TIME

You may want to let students know the following about MEAs:

- MEAs are longer problems; there are no immediate answers. Instead, students should expect to work on the problem and gradually revise their solution over a period of 45 minutes to an hour.
- MEAs often have more than one solution or one way of thinking about the problem.
- Let the students know ahead of time that they will be presenting their solutions to the class. Tell them to prepare for a 3-5 minute presentation, and that they may use overhead transparencies or other visuals during their presentation.
- Let the students know that you won't be answering questions such as "Is this the right way to do it?" or "Are we done yet?" You can tell them that you will answer clarification questions, but that you will not guide them through the MEA.
- Remind students to make sure that they have returned to the problem statement to verify that they have fully answered the question.
- If students struggle with writing the letter, encourage them to read the letter out loud to each other. This usually helps them identify omissions and errors.

OBSERVING STUDENTS AS THEY WORK ON THE Trash Trouble MEA

You may find the Observation Form (page 16) useful for making notes about one or more of your teams of students as they work on the MEA. We have found that the form could be filled out "real-time" as you observe the students working or sometime shortly after you observe the students. The form can be used to record observations about what concepts the students are using, how they are interacting as a team, how they are organizing the data, what tools they use, what revisions to their solutions they may make, and any other miscellaneous comments.

PRESENTATION FORM (Optional)

As the teams of students present their solutions to the class, you may find it helpful to have each student complete the presentation form on page 17. This form asks students to evaluate and provide feedback about the solutions of at least two teams. It also asks students to consider how they would revise their own solution to the Trash Trouble MEA after hearing of the other teams' solutions.

STUDENT REFLECTION FORM

You may find the Student Reflection Form (page 18) useful for concluding the MEA with the students. The form is a debriefing tool, and it asks students to consider the concepts that they used in solving the MEA and to consider how they would revise their previous solution after hearing of all the different solutions presented by the various teams. Students typically fill out this form after the team presentations.

STANDARDS ADDRESSED

NCTM MATHEMATICS STANDARDS

Numbers and Operations:

- Work flexibly with fractions, decimals, and percents to solve problems
- Understand and use ratios and proportions to represent quantitative relationships
- Judge the reasonableness of numerical computations and their results

Algebra

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Relate and compare different forms of representation for a relationship
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use symbolic algebra to represent and explain mathematical relationships
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
- Draw reasonable conclusions about a situation being modeled

- Develop an initial conceptual understanding of different uses of variables
- Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships

Data Analysis and Probability

- Find, use, and interpret measures of center and spread, including mean and interquartile range
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots

Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

- Develop and evaluate mathematical arguments and proofs

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Use representations to model and interpret physical, social, and mathematical phenomena

NRC Science Standards

Inquiry

- Use appropriate tools and techniques to gather, analyze and interpret data
- Develop descriptions, explanations, predictions, and models using evidence
- Think critically and logically to make the relationships between evidence and explanations
- Recognize and analyze alternative explanations and predictions
- Communicate scientific procedures and explanations
- Use mathematics in all aspects of scientific inquiry

NATURAL HAZARDS

- Human activities also can induce hazards through resource acquisition, urban growth, land-use decisions, and waste disposal. Such activities can accelerate many natural changes.
- Natural hazards can present personal and societal challenges because misidentifying the change or incorrectly estimating the rate and scale of change may result in either too little attention and significant human costs or too much cost for unneeded preventive measures.

SCIENCE AND TECHNOLOGY IN SOCIETY

- Science influences society through its knowledge and worldview. Scientific knowledge and the procedures used by scientists influence the way many individuals in society think about them- selves, others, and the environment. The effect of science on society is neither entirely beneficial nor entirely detrimental.

NATURAL RESOURCES

- Humans use many natural systems as resources. Natural systems have the capacity to reuse waste, but that capacity is limited. Natural systems can change to an extent that exceeds the limits of organisms to adapt naturally or humans to adapt technologically.

ENVIRONMENTAL QUALITY

- Natural ecosystems provide an array of basic processes that affect humans. Those processes include maintenance of the quality of the atmosphere, generation of soils, control of the hydrologic cycle, disposal of wastes, and recycling of nutrients. Humans are changing many of these basic processes, and the changes may be detrimental to humans.
- Materials from human societies affect both physical and chemical cycles of the earth.
- Many factors influence environmental quality. Factors that students might investigate include population growth, resource use, population distribution, overconsumption, the capacity of technology to solve problems, poverty, the role of economic, political, and religious views, and different ways humans view the earth.

Common Core State Standards

- 5 MD-2: represent and interpret data

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A* *received, candidate C received nearly three votes."*

6.RP.2 Understand the concept of a unit rate *a*/*b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

- A-CED-2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- F-IF-6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
- F-LE 1: Construct and compare linear, quadratic, and exponential models and solve problem
    1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - S-ID 6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
      a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
      b) Informally assess the fit of a function by plotting and analyzing residuals.
      c) Fit a linear function for a scatter plot that suggests a linear association.
- S-ID 7: Interpret linear models. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- S-IC 1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-IC 2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Standards for Mathematical Practices integration with MEAs

Mathematical Practice	How it occurs in MEAs
1. Make sense of problems and persevere in solving them.	As participants work through iterations of their models they continue to gain new insights into ways to use mathematics to develop their models. The structure of MEAs allows for participants to stay engaged and to have sustained problem solving experiences.
2. Reason abstractly and quantitatively	MEAs allow participants to both contextualize, by focusing on the real world context of the situation, and decontextualize by representing a situation symbolically.
3. Construct viable arguments and critique the reasoning of others.	Throughout MEAs while groups are working and presenting their models.
4. Model with mathematics.	This is the essential focus of MEAs; for participants to apply the mathematics that they know to solve problems in everyday life, society, or the workplace. This is done through iterative cycles of model construction, evaluation, and revision.
5. Use appropriate tools strategically.	Materials are made available for groups as they work on MEAs including graph paper, graphing calculators, computers, applets, dynamic software, spreadsheets, and measuring devices.
6. Attend to precision.	Precise communication is essential in MEAs and participants develop the ability to communicate their mathematical understanding through different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.
7. Look for and make use of structure.	Participants in MEAs can use their knowledge of mathematical properties and algebraic expressions to develop their solutions.
8. Look for and express regularity in repeated	As participants develop their models the patterns they notice

reasoning.	can assist in their model development.

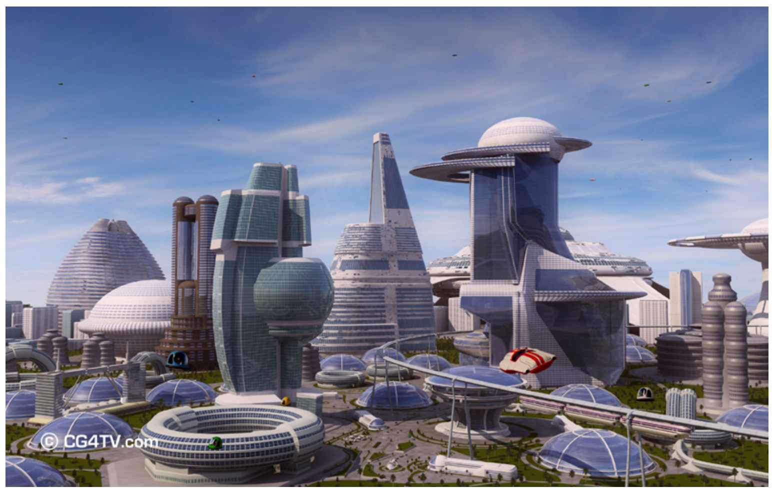
What Will the Future Hold? How Math Can Help People Make Predictions.

What do you want to know about the future? People in any time period have been interested in what the future will be like. Movies have depicted time travel and the allure of knowing future information.   One of the powers of mathematical knowledge and understanding is the ability to recognize patterns and predict. Recent news article titles have shown the wide range of math's predictive power. Can math help to predict future terrorist attacks? Can math help predict NFL placement? Simple math can help predict your weight loss. Math can predict marriage success. Can mathematics predict disease? Can mathematics predict the future?

Making a mathematical prediction is more than just guessing. Predictions without a basis in math are little more than guessing and guessing is not reliable. If you took a math test and did not try to actually figure out the problems but just guessed the odds are overwhelming that you would be wrong the majority of the time. So, it becomes important to actually figure out the proper answer to a question in the same way you need to use math as a basis for making proper predictions.

How do predictions work in a practical manner? They work in the sense that predictions are based upon a careful analysis of patterns, which are essentially recurring events. Patterns can be found in languages, sciences, and even the arts. Patterns exist in a number of ways including in weather, sports, finances, population, traffic, and more.

It is important to understand that math is more than formulas, symbols, or equations but it involves reasoning, sense making, communication, and real world connections. It is using your mind to solve problems that affect a large number of people. Financial advisors make predictions based on what they believe will happen to the stock market. The predictions are based on collected data and patterns. Can math provide predictions that possess 100% accuracy? No, it would be next to impossible to always make 100% accurate predictions. But, a prediction based on solid data can allow you to make an assumption that is close to the likelihood of accuracy and this is clearly very valuable.



Readiness Questions

1. What would you want to know about the future?

2. What is a mathematical prediction?

3. What are some examples of areas where math is used to predict?

4. What are the difficulties associated with making predictions?

5. What can happen if a prediction is wrong?

Environmental Protection Agency (EPA) Concerned about United States Trash Production

As the amount of trash produced each year increases solutions have to be found for where to store the trash and how it can be reused. Some landfills are closing because they are full. Some statistics on trash can be surprising.

- Almost one third of the trash produced each year is packaging materials.
- Americans throw away nearly 2.5 million plastic bottles every hour.
- An additional five million tons of trash is generated during the holidays. Four million of those tons are wrapping paper and shopping bags.
- Each year Americans throw away enough office paper to build a twelve-foot wall from Los Angeles to New York City.

Problem Statement:

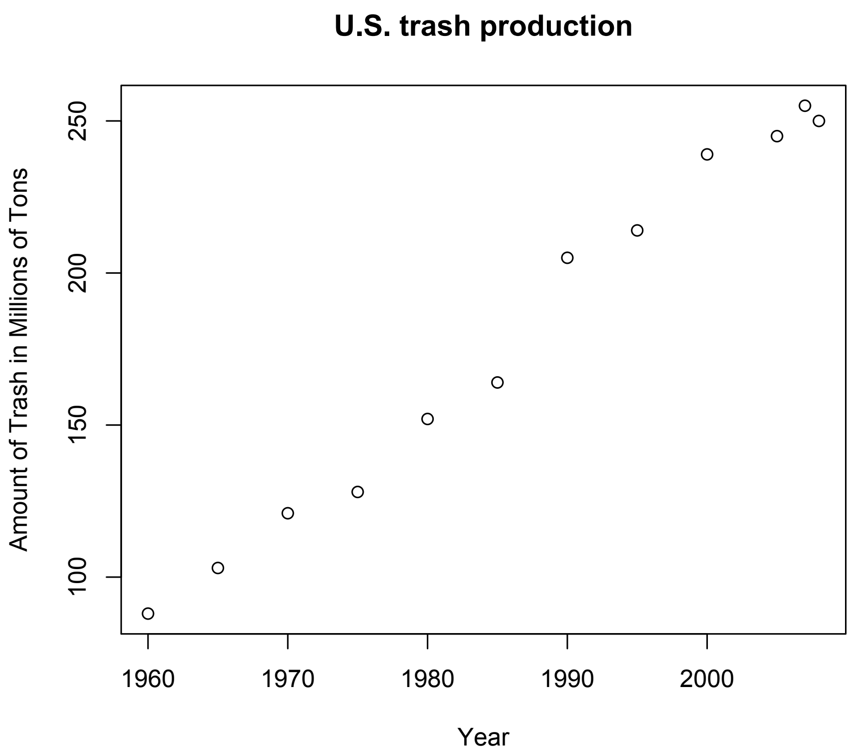
In order to make plans and prepare for what to do with America's trash it is important to have an understanding of how much trash there will be. The EPA would like your help in determining how much trash the United States will produce in 2020. Use the data on the next page to help you make your prediction. You will write a letter to the EPA that gives your prediction for the amount of trash that the U.S. will produce in 2020 and also describes your procedure for how you came up with your prediction. Your procedure should be described so that it could be used in future years when new data about U.S. trash production is available. You can also include suggestions for how the amount of trash produced each year could be lowered.

U.S Trash Production and Recycling Rates

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2008	2013
Amount of trash (million tons)	88	103	121	128	152	164	205	214	239	245	250	254
U.S. Population (in millions)	179	191	203	215	227	238	250	266	282	300	304	317
Percent of trash recycled	6.4	6.2	6.6	7.3	9.6	10.1	16.2	26	29	31.7	33.2	34.3

Note:

Graph paper and rulers should be available if student groups want to use them. The graph below is provided as a reference for teachers.



References for background article:

http://en.wikipedia.org/wiki/Numb3rs

http://www.mathworksheetscenter.com/mathtips/mathandpredictions.html

References for problem client statement

More detailed information about trash (municipal solid waste) production and recycling can be found in the document below.

http://www.epa.gov/wastes/nonhaz/municipal/pubs/msw2008rpt.pdf

http://www.isical.ac.in/~sdebasis/linmodel/datasets.html

http://www.cleanair.org/Waste/wasteFacts.html

More linear data can be found at:

http://illuminations.nctm.org/LessonDetail.aspx?id=L298 http://www.isical.ac.in/~sdebasis/linmodel/datasets.html

http://www.censusscope.org/us/chart_popl.html

http://www.google.com/publicdata?ds=uspopulation&met=population&tdim=true&dl=en&hl=en&q=United+States+population

If students use regression equations an important statistical point to make with students is that it is not legitimate to extrapolate using values outside the domain of values for the independent variable when using a regression model because it leads to greater uncertainty. In this case the data was for the years 1960 to 2008. From a statistical perspective only predictions should be made for the amount of trash in this time period. Predictions can be made with a linear regression equation beyond 2008, but many factors could affect what happens in the future that could change the prediction.

This table could be given to students instead of using the percent of trash recycled if you would rather have the students just have the number of tons recycled.

U.S Trash Production and Recycling Rates

Year	Amount of trash (million tons)	U.S. Population (In millions)	Millions of tons recycled
1960	88	179	5.6
1965	103	191	6.5
1970	121	203	8.0
1975	128	215	9.3
1980	152	227	14.5
1985	164	238	16.7
1990	205	250	33.2
1995	214	266	58
2000	239	282	69.4
2005	245	300	79.2
2008	250	304	82.9
2013	254	317	87

OBSERVATION FORM FOR TEACHER- Trash Trouble MEA

Team: ___________________________________

Science, Technology, Engineering, and Mathematics (STEM) Concepts Used:

What STEM concepts and skills did the students use to solve the problem?

Team Interactions:

How did the students interact within their team or share insights with each other?

Data Organization & Problem Perspective:

How did the students organize the problem data? How did the students interpret the task? What perspective did they take?

Tools:

What tools did the students use? How did they use these tools?

Miscellaneous Comments about the team functionality or the problem:

Cycles of Assessment & Justification:

How did the students question their problem-solving processes and their results? How did they justify their assumptions and results? What cycles did they go through?

PRESENTATION FORM – Trash Trouble MEA

Name____

While the presentations are happening, choose TWO teams to evaluate. Look for things that you like about their solution and/or things that you would change in their solution. You are not evaluating their style of presenting. For example, don't write, "They should have organized their presentation better." Evaluate their solution only.

Team _______________________________

What I liked about their solution:

What I didn't like about their solution:

Team _______________________________

What I liked about their solution:

What I didn't like about their solution:

After seeing the other presentations, how would you change your solution? If you would not change your solution, give reasons why your solution does not need changes.

STUDENT REFLECTION FORM – Trash Trouble MEA

Name ______________________________ Date______________________________

1. What mathematical or scientific concepts and skills (e.g. ratios, proportions, forces, etc.) did you use to solve this problem?

2. How well did you understand the concepts you used?

Not at all A little bit Some Most of it All of it

Explain your choice:

3. How well did your team work together? How could you improve your teamwork?

4. Did this activity change how you think about mathematics?